Power-law statistics and stellar rotational velocities in the Pleiades

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Abstract

In this paper we will show that, the non-gaussian statistics framework based on the Kaniadakis statistics is more appropriate to fit the observed distributions of projected rotational velocity measurements of stars in the Pleiades open cluster. To this end, we compare the results from the κ and q-distributions with the Maxwellian.

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I. INTRODUCTION

Some restrictions to the applicability of the statistical mechanics have motivated the investigation of the power-law or non-gaussian statistics, both from theoretical and experimental viewpoints. In this concern, the nonextensive statistical mechanics [1] and extensive generalized power-law statistics [2] are the most investigated frameworks. In the later one, recent efforts on the kinetic foundations of the κ-statistics leads to a power-law distribution function and the κ-entropy which emerges in the context of the special relativity and in the so-called *kinetic interaction principle* (see Ref. [2]). Formally, the κ-framework is based on κ-exponential and κ-logarithm functions, which are defined by

$$\exp_\kappa(f) = (\sqrt{1+\kappa^2 f^2} + \kappa f)^{1/\kappa}, \tag{1}$$

$$\ln_\kappa(f) = \frac{f^\kappa - f^{-\kappa}}{2\kappa}, \tag{2}$$

whereas the κ-entropy associated with the κ-statistics is given by [2]

$$S_\kappa = -\int d^3p f \ln_\kappa f = -\langle \ln_\kappa(f) \rangle. \tag{3}$$

The expressios above reduces to the standard results in the limit $\kappa = 0$.

The Tsallis statistics has been investigated in a wide range of problems in physics [1]. In the astrophysical domain, the first applications of this powerlaw statistics studied stellar polytropes [3] and the peculiar velocity function of galaxy clusters [4]. More recently, Kaniadakis statistics has also been studied in the theoretical and experimental context [12], however the first application with a possiple connection with astrophysical system has been the simulation in relativistc plasmas. In this regard, the powerlaw energy distribuition provides a strong argument in favour of the Kaniadakis statistics [5]. Therefore, in such systems where nonextensivity holds, the classical statistics may be generalized. In this sense, one of the most puzzling questions in stellar astrophysics in the past 50 years is that concerning the nature of the statistical law controlling the distribution of stellar rotational velocity, in spite of the large acceptance that stellar rotation axes have a random orientation [6]. In the middle of the past century, Chandrasekhar and Munch [7] were the first to derive analytically the distribution of stellar projected rotational velocity, on the basis of a Gaussian

[1] For a complete and updated list of refences see http://tsallis.cat.cbpf.br/biblio.htm

distribution. In their approach, these authors first assumed a parametric form for a function $f(v)$, where v is the true rotational velocity, then computed the corresponding distribution of the projected rotational velocity $Vsini$ and finally adjusted a set of stellar parameters to reproduce the $Vsini$ measurements. Two decades later, Deutsch [8] claimed that the distribution of stellar rotational velocities should have the form of a Maxwellian-Boltzmann law. Nevertheless, a number of studies have shown a clear discrepancy between theory and observations, where observed distributions are not fitted by a Gaussian or Maxwellian function with a good level of significance. A Gaussian or Maxwellian distribution that fits the fast rotators fails to account for low rotation rates. On the other hand, a fit to slow rotators fails to explain the rapidly rotating stars [9, 10].

Rotation is one of the most important observable in stellar astrophysics, driving strongly the evolution of stars, providing also valuable informations on stellar magnetism, mixing of chemical abundances in stellar interior, tidal interaction in close binaries, and engulfing of brown dwarfs and planets. In addition, if the present value of the rotational velocity of stars at a given evolutionary stage reflects the original angular momentum with which they were formed, the behaviour of the distribution of rotational velocity may also be used to study some of the characteristics of the physical processes controlling star formation. Early studies on the nature of the statistics controlling the distribution of stellar rotational velocity were based on $Vsini$ measurements with poor precision, which, admittedly, can lead to systematic errors on the final analyses for low $Vsini$ values. Here, we show that the question of the nature of the distribution of stellar rotational velocity, at least for low-mass stars in the Pleiades open cluster, is not simply a question of which mathematical function model is used, but it depends primarily on the statistical mechanics applied, which should be general enough to take into account the changes in rotation with time.

In this work we have investigated the effects of the powerlaw statistics on the observed distribution of projected rotational velocity measurements of stars in the Pleiades open cluster by considering a κ-distribution functions. In this regard, it is worth emphasis that earlier study based on the Tsallis statistics has been developed in Ref. [16]. However, in the present work, to study the effects of the powerlaw statistics on the observed distribution in the Pleiades open cluster we use the more recent generalization of Kaniadakis [2] and, for completeness, we compare the results with the ones obtained in the context of the Tsallis statistics. This paper is organized as follows. In Sec II, based on the basic formalism

presented in Ref. [8], we present a generalization of the rotational velocity distribution in the spirit of Kaniadakis statistics. A brief discussion on the stellar sample is made in Sec. III. Our main results are discussed in Sec. IV and we summarize the main conclusions in Sec. V.

II. κ-DISTRIBUTION FUNCTION

As well know, a large portion of the experimental evidence, as well as some theoretical considerations supporting Tsallis and Kaniadakis proposal involves a non-Maxwellian (powerlaw) distribution function associated with the thermostatistical description from the variety of the physical systems [11, 12]. In Tsallis framework, the equilibrium velocity q-distribution may be derived from at least three different methods, namely: (i) through a simple nonextensive generalization of the Maxwell ansatz, which is based on the isotropy of the velocity space [13]; (ii) within the nonextensive canonical ensemble, that is, maximizing Tsallis entropy under the constraints imposed by normalization and the energy mean value [14] and (iii) using a more rigorous treatment based on the nonextensive formulation of the Boltzmann H-theorem [15]. Here, we revisit the first method by considering the Kaniadakis statistical which is based on $\exp_\kappa$ and $\ln_\kappa$ given by Eq. (1), where f is a function of random variables that includes the standard exponential as the limiting case when $\kappa \to 0$.

It is widely known that Deutsch [8] has considered the distribution function for the magnitude of a vector that has random orientation. For this, it is required to find the distribution function of a positive scalar ω, which is the magnitude of a vector $\vec{\omega}$. We assume that the distribution of $\vec{\omega}$ is isotropic. We also assume that if it is decomposed into components along Cartesian axes, the distribution of any component is independent of the other components. Deutsch has defined Ω as the non-dimensional quantity ωj, where j is a parameter with the dimension ω^{-1}, so

$$\vec{\Omega} = \Omega_x \vec{i} + \Omega_y \vec{j} + \Omega_z \vec{k} \tag{4}$$

The probability that Ω_x lies in the interval $[\Omega_x; \Omega_x + d\Omega_x]$, Ω_y in $[\Omega_y; \Omega_y + d\Omega_y]$ and Ω_z in $[\Omega_z; \Omega_z + d\Omega_z]$ is then

$$F(\Omega)d^3\Omega = f(\Omega_x)f(\Omega_y)f(\Omega_z)d\Omega_x d\Omega_y d\Omega_z \tag{5}$$

where $\Omega = \sqrt{\Omega_x^2 + \Omega_y^2 + \Omega_z^2}$ and $F(\Omega)$ is the standard Maxwellian distribution function

$$F(\Omega) = \frac{4}{\sqrt{\pi}} \Omega^2 \exp\left(-\Omega^2\right) \tag{6}$$

Let us now consider the arguments given in Refs. [13, 16]. One can modify the basic hypothesis of statistical independence between the distributions associated with the components of $\vec{\Omega}$, based on the κ-statistics. As pointed out in [13] and [16], the independence between the three velocity components does not hold in systems with long-range interaction, or statiscally correlated, where the power-law statistics character is observed. Taking such arguments into account, the generalization for Eq. (6) in the light from Kaniadakis the framework reads,

$$F(\Omega) d^3\Omega = \exp_\kappa(\ln_\kappa f(\Omega_x) + \ln_\kappa f(\Omega_y) + \ln_\kappa f(\Omega_z)) d\Omega_x d\Omega_y d\Omega_z, \tag{7}$$

where the κ-exponential and κ-logarithm are given by identities (1) and (2). In particular, in the limit $\kappa = 0$ the standard expression (5) is recovered. Note also that $\ln_\kappa(\exp_\kappa(f)) = \exp_\kappa(\ln_\kappa(f)) = f$, and $\frac{d \ln_\kappa f}{dx} = \frac{f^\kappa + f^{-\kappa}}{2f} \frac{df}{dx}$ are satisfied. Therefore, the partial differentiation of the κ-ln of (7) with respect to Ω_i leads to

$$\frac{\partial \ln_\kappa F}{\partial \Omega_i} = \frac{\partial}{\partial \Omega_i} (\ln_\kappa f(\Omega_x) + \ln_\kappa f(\Omega_y) + \ln_\kappa f(\Omega_z)), \tag{8}$$

or, equivalently,

$$\frac{F^\kappa + F^{-\kappa}}{2F} \frac{1}{\chi} \frac{dF}{d\chi} = \frac{1}{\Omega_i} \frac{d}{d\Omega_i} \ln_\kappa f_i, \tag{9}$$

where $\chi = \sqrt{\Omega_x^2 + \Omega_y^2 + \Omega_z^2}$. Now, by defining

$$\phi(\chi) = \frac{F^\kappa + F^{-\kappa}}{2F} \frac{1}{\chi} \frac{dF}{d\chi}, \tag{10}$$

we may rewrite (9) as

$$\phi(\chi) = \frac{1}{\Omega_i} \frac{d}{d\Omega_i} \ln_\kappa f_i \tag{11}$$

The second member of the above equation depends only on Ω_i, with $i = x, y, z$. Hence, equation (11) can be satisfied only if all its members are equal to one and the same constant, not depending on any of the velocity components. Thus, we can make $\phi(\chi) = -2/\sigma_\kappa^2$, where the parameter σ_κ is the width of the κ-distribution, leading to

$$\frac{1}{\Omega_i} \frac{\partial}{\partial \Omega_i} (\ln_\kappa f_i) = -\frac{2}{\sigma_\kappa^2}. \tag{12}$$

Hence, the solutions of Eq. (12) for $f(\Omega_i)$ is given by the κ-distribution

$$f(\Omega_i) = \exp_\kappa(-\Omega_i^2/\sigma_\kappa^2) \tag{13}$$

From (13) we see that the Gaussian probability curve is replaced by the characteristic powerlaw behavior of Kaniadakis framework and, as expected, the limit $\kappa = 0$ recovers the exponential result. Note also that for any values of the κ-parameter, the powerlaw (13) does not exhibits a cut-off in the maximal allowed rotational velocities, and it is straightforward to show that $F(\Omega)$ is given by

$$F(\Omega) = \exp_\kappa(-\Omega^2/\sigma_\kappa^2) \tag{14}$$

The probability of finding ω in the interval $[\Omega, \Omega+d\Omega]$ is determined by $\Psi(\Omega) = \int f(\Omega)d^3\Omega$ which leads to a power law that belongs to the same class of power law as given in Eq. (13), i.e.,

$$\Psi(\Omega) = 4\pi\Omega^2 \exp_\kappa(-\Omega^2/\sigma_\kappa^2). \tag{15}$$

Here, it is worth mentioning that the standard distribution of the true rotational velocity V for a star sample is $F(V) \sim V^2 \exp(-V^2)$. As shown by Deutsch [8], the standard observed distribution of the projected rotational velocity $V sini$, for a random orientation of axes, must be given by $\phi(y) \sim y \exp(-y^2)$ [17], with $y = V sini$. Henceforth, the κ-distribution $\phi_\kappa(y)$ should reproduce the standard one, in the same way as $F_\kappa(v)$ recovers $F(V)$ in the $\kappa = 0$ limiting case. Therefore, by considering this arguments, we introduce the following distribution function for the observed stellar rotational velocities

$$\phi_\kappa(y) = y \exp_\kappa(-y^2/\sigma_\kappa^2). \tag{16}$$

III. THE STELLAR SAMPLE

The rotational velocities $V \sin i$ used in the present analysis were taken from the rotational survey for the Pleiades stars carried out by Queloz et al. [18]. We have selected 219 stars from the original sample given by those authors. All the selected objects are low-mass stars and provide a complete and unbiased rotation data set for stars in the $B - V$ range $(0.4-1.4)$ corresponding to an effective temperature range from 4000 K to 6000 K and a mass range from $0.6M_\odot$ to $1.2M_\odot$. For a complete discussion on the observational procedure,

TABLE I: Best values of the parameters of Kaniadakis (κ and σ_κ) and Tsallis (q and σ_q) distribution determined using Kolmogorov–Smirnov test for the rotational velocity of stars in the Pleiades cluster.

$\Delta(B-V)$	N	κ	σ_κ	P_{max}	q	σ_q	P_{max}
0.40–1.40	217	$0.446^{+0.048}_{-0.073}$	7.81	0.28	$1.334^{+0.038}_{-0.055}$	6.93	0.23

calibration and error analysis the reader is referred to Queloz et al. [18]. We should observe that individual errors in $V\sin i$ measurements are better than about 1 km/s and should not play a significant role on the observed distributions.

If we plot the observed distribution we note that there are two stars with velocity (105 km/s and 160 km/s) too far from the peak of the distribution (5 – 8 km/s). We have, therefore, excluded these stars with exceedingly high value of $V\sin i$ from the sample.

In order to avoid biases due to arbitrary choices of bin range when constructing the frequency histograms, we have decided to study the observed cumulative distribution of the rotational velocities, $V\sin i$, and compare it with the integral of the probability distribution function in (16). The normalized cumulative distribution is given by

$$\Sigma_\kappa(y) = \frac{\int_0^y y\exp_\kappa(-y^2/\sigma_\kappa^2)dy}{\int_0^\infty y\exp_\kappa(-y^2/\sigma_\kappa^2)dy}. \tag{17}$$

IV. RESULTS

Parallel with the Kaniadakis distribution we have also fitted the data to the Tsallis distribution in order to evaluate which one would best fit the observations. The Tsallis generalized maxwellian is a two parameter (q and σ_q) function given by

$$\phi_q(y) = y\left[1-(1-q)\frac{y^2}{\sigma_q^2}\right]^{1/(1-q)}. \tag{18}$$

In the limit $q=1$ the standard Maxwellian is recovered.

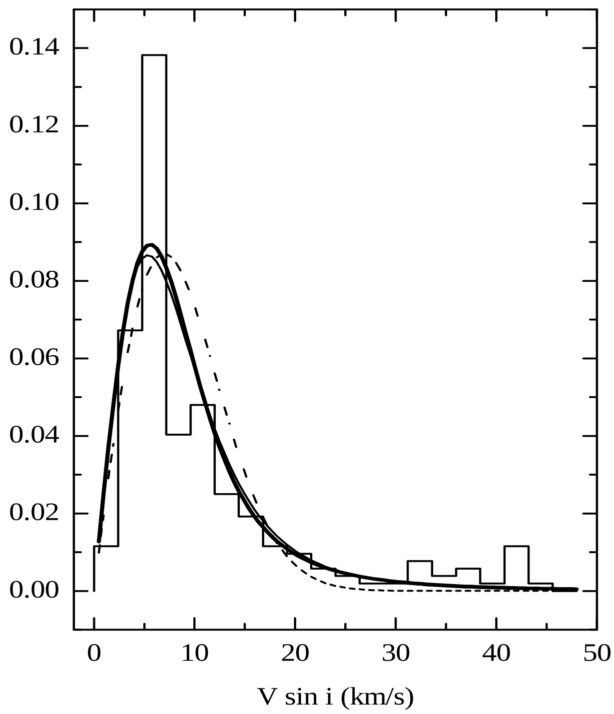

FIG. 1: Observed distribution (histogram) of the rotational velocity of the stars in the Pleiades open cluster. The curves represent the best fitted maxwellian (dashed line), Tsallis (thin line) and Kaniadakis (thick line) distribution. The fitting parameters are in Table 1.

To calculate the best values of the distributions parameters for the observed cumulative distribution we used the Kolmogorov-Smirnov statistical test.

The distribution functions were used to fit the observational data, to obtain the best $\phi_\kappa(y)$ and $\phi_q(y)$, giving the best κ and q-value together with the best σ_κ and σ_q for the corresponding distribution. The results are shown in Table 1. We can clearly see that the $V \sin i$ distribution of the Pleiades stars do not obey a standard Maxwellian function since the values of κ and q are significantly different from 0 and 1, respectively.

Fig. 1 shows the best fits for the histogram of the observed distribution of $V \sin i$ according the results in Table 1. The κ and q-Maxwellian functions are represented, respectively by the thick ($\kappa = 0.45$) and thin ($q = 1.33$) lines. The dashed line represents standard Maxwellian function. The distribution of observed $V \sin i$ is without a doubt more adequately fitted

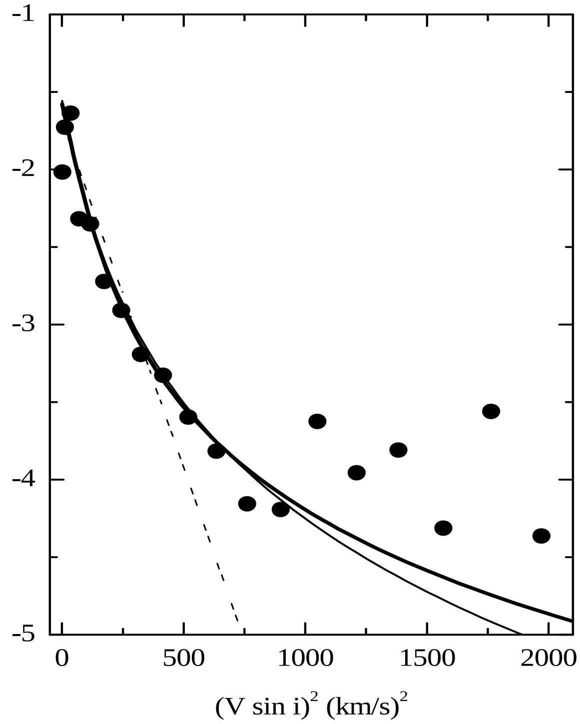

FIG. 2: As in Fig. 1 but for the logarithm of the distribution function divided by $V \sin i$ as a function of $(V \sin i)^2$.

by either the κ or the q-Maxwellian function. This is more noticeable in Fig. 2 where we have plotted the logarithm of the distribution divided by $V \sin i$, that is $log(\phi(y)/y)$, as a function of $(V \sin i)^2$ so that the standard Maxwellian is represented by a straight (dashed) line. The Kaniadakis distribution is represented by the thick line while Tsallis distribution by the thin line. We observe that none of the distribution fit well the high velocity end of the observed data. Although the Kaniadakis function fits the data slightly better than Tsallis function, the difference may be regarded as marginal as indicated by the values of the maximum probability of the Kolmogorov-Smirnov statistical test (Table 1).

Finally, in Fig. 3 we present the behaviour the parameter σ, representing the width of the two non-Maxwellian, as a function of the parameter κ and q. It is clear that, at least for the present stellar sample, the standard Maxwellian ($\kappa = 0$ or $q = 1$) is in the rejection region, i.e., outside the curve which delineates 0.05 significance level.

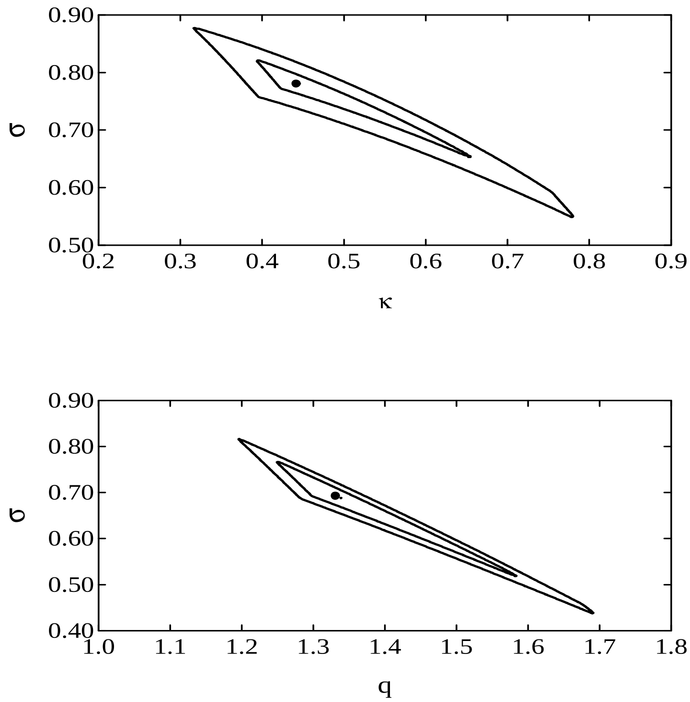

FIG. 3: Rejection region (outside curves) of the null hypothesis that the $V \sin i$ distribution is drawn from the κ-distribution (upper panel) at 0.05 and 0.15 confidence level and from q-distribution (lower panel) at 0.05 and 0.10 confidence level. The dot ($\bullet$) represents the maximum probability for the pair ($\kappa - \sigma_\kappa$) and ($q - \sigma_q$).

V. CONCLUSIONS

In this work we have used non-gaussian statistics to investigate the observed distribution of projected rotational velocity of stars in the Pleiades open cluster. We have studied in details the Kaniadakis distribution and shown that it fits more closely the observed distribution than the standard maxwellian. A comparison with the Tsallis statistics shows that, at least when the observed distribution presents an extended tail, as it is the case of rotational velocity of stars in the Pleiades, both distributions give equivalent, though not entirely satisfactory results.

As discussed in Sec. IV, the best fits for the histogram of observed distribution are non-

gaussian with $\kappa = 0.45$ and $q = 1.33$ for Kaniadakis and Tsallis parameter, respectivily. In particular, we emphasize that the result of the q-parameter is consistent with the upper limit $q < 2$ obtained from several independents investigations in the quantum limit [19] and in the non-quantum limit [20].

Finally, it is worth mentioning that the the best fits in κ and q can be recalculated by considering a more robust stellar sample. In this respect, the stellar radial velocity of a sample of open clusters are being studied. This issue will be addressed in a forthcoming communication.

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[1] C. Tsallis, J. Stat. Phys. **52**, 479 (1988); S. R. A. Salinas, C. Tsallis (Eds.), Braz. J. Phys. (special number) **29** (1999); S. Abe, Y. Okamoto (Eds.), Nonextensive Statistical Mechanics and its Applications, Springer, Heidelberg (2001); M. Gell-Mann and C. Tsallis (Eds.), Nonextensive Entropy - Interdisciplinary Applications, Oxford University Press, New York, (2004).

[2] G. Kaniadakis, Physica A **296**, 405 (2001); G. Kaniadakis, Phys. Rev. E **66**, 056125 (2002); G. Kaniadakis, Phys. Rev. E **72**, 036108 (2005).

[3] A. Plastino, A. R. Plastino, Phys. Lett. A **174**, 384 (1993).

[4] A. Lavagno, G. Kaniadakis, M. Rego-Monteiro, P. Quariti, C. Tsallis, Astrophys. Lett. Commun. **35**, 449 (1998).

[5] G. Lapenta, S. Markidis, A. Marocchino and G. Kaniadakis, Astrophys. Journ. **666**, 949 (2007).

[6] O. Struve, Pop. **53**, 202 (1945).

[7] S. Chandrasekhar, G. Munch, Astrophys. Journ. **111**, 142 (1950).

[8] A. J. Deutsch, in: A Slettebak (Ed.), Stellar Rotation, IAU Colloquium, Reidel, Dordrecht, 1970, p. 207.

[9] M. M. Dworetssky, Astrophys. Journ. **28**, 101 (1974)

[10] S. C. Wolff, S. Edwards and G. W. Preston, Astrophys. J. 252 (1982) 322.

[11] J. A. S. Lima, R. Silva and J. Santos, Phys. Rev E **61**, 3260 (2000); E. M. F. Curado, F. D. Nobre, Phys. Rev. E **67**, 021107 (2003); M. Shiino, Phys. Rev. E **67**, 056118 (2003); B. Dybiec and E. Gudowska-Nowak, Phys. Rev E **69**, 016105 (2004); R. Silva and J. S. Alcaniz, Physica A **341**, 208 (2004); ibdem Phys. Lett. A **313**, 393 (2003); Du Jiulin, Europhys. Lett. **67**, 893 (2004); T. S. Biro, and A. Jakovav, Phys. Rev. Lett. **94**, 132302 (2005); T. S. Biro, and G. Purcsel, Phys. Rev. Lett. **95**, 162302 (2005); T. Kodama, H,-T. Elze, C. E. Aguiar, and T. Koide, Europhys. Lett. **70**, 439 (2005); M. S. Reis, V. S. Amaral, R. S. Sarthour and I. S. Oliveira Phys. Rev. B **73**, 092401 (2006); P. Douglas, S. Bergamini, and F. Renzoni, Phys. Rev. Lett. **96**, 110601 (2006); R. Silva, G. S. França, C. Vilar, and J. S. Alcaniz, Phys. Rev. E **73**, 026102 (2006); J.C. Carvalho, B.B. Soares, B.L. Canto Martins, J.D. do Nascimento Jr., A. Recio-Blanco and J.R. De Medeiros, Physica A **384**, 507 (2007); F. I. M. Pereira, R. Silva, and J. S. Alcaniz Phys. Rev. C **76**, 015201 (2007); Bin Liu, J. Goree, Phys. Rev. Lett. **100**, 055003 (2008).

[12] S. Abe, G. Kaniadakis, and A. M. Scarfone, J. Phys. A: Math. Gen. **37**, 10513 (2004); G. Kaniadakis, Phys. Lett. A **288**, 283 (2001); G. Kaniadakis and A. M. Scarfone, Physica A, **305**, 69 (2002); G. Kaniadakis, P. Quarati and A. M. Scarfone, Physica A, **305**, 76 (2002); G. Kaniadakis and A. M. Scarfone, Physica A, **340**, 102 (2004); G. Kaniadakis, M. Lissia and A. M. Scarfone, Physica A **340**, 41 (2004); G. Kaniadakis, M. Lissia and A. M. Scarfone, Phys. Rev. E **71**, 046128 (2005); A. M. Teweldeberhan, H. G. Miller and R. Tegen, Int. J. Mod. Phys. E **12**, 669 (2003); A. Rossani and A. M. Scarfone, J. Phys. A: Math. Gen. **37**, 4955 (2004); Cravero, G. Iabichino, G. Kaniadakis, E. Miraldi and A. M. Scarfone, Physica A **340**, 410 (2004); D. Rajaonarison, D. Bolduc and H. Jayet, Econ. Lett. **86**, 13 (2005); A. M. Scarfone, Phys. Rev. E **71**, 051103 (2005); R. Silva, Phys. Lett. A **352**, 17 (2006); Eur. Phys. J. B **54**, 499 (2006); J. M. Silva, R. Silva, J. A. S. Lima Phys. Lett. A **372**, 5754 (2008).

[13] R. Silva, A. R. Plastino and J. A. S. Lima, Phys. Lett. A, **249**, 401 (1998).

[14] S. Abe, Physica A **269**, 403 (1999).

[15] J. A. S. Lima, R. Silva and A. R. Plastino, Phys. Rev. Lett. **86**, 2938 (2001).

[16] B. B. Soares, J.C. Carvalho, J.D. do Nascimento Jr. and J.R. De Medeiros, Physica A **364**, 413 (2006).

[17] R. P. Kraft, in *Spectroscopic Astrophysics. An Assessment of the Contributions of Otto Struve*,

G. H. Herbig, ed. (Berkeley: University of California Press), 1970, p.385.

[18] D. Queloz, S. Allain, J.-C. Mermilliod, J. Bouvier and M. Mayor, Astron. and Astrophys. **335**, 183 (1998).

[19] S. Abe and A. K. Rajagopal, Phys. Rev. Lett. **12**, 120601 (2003).

[20] B. M. Boghosian, Braz. Journ. Phys. **29**, 91 (1999); S. H. Hansen, D. Egli, L. Hollenstein and C. Salzmann, New Astronomy **10**, 379 (2005); R. Silva and J. A. S. Lima, Phys. Rev. E **72**, 057101 (2005), C. Papadimitriou, M. Kalimeri and K. Eftaxias, Phys. Rev. E **77**, 036101 (2008).